18.06 (Fall '12) Problem Set 1

This problem set is due Thursday, September 13, 2012 by 4pm in 2-255. The problems are out of the 4th edition of the textbook. For computational problems, please include a printout of the code with the problem set (for MATLAB in particular, `diary(“filename”)` will start a transcript session, `diary off` will end one, also copy and paste usually work as well.)

1. Do Problem 21 from 2.1.

2. Do Problem 21 from 2.2.

3. Do Problem 17 from 2.3.

4. Do Problem 22 from 2.4.

5. Do Problem 11 from 2.5.

6. Do Problem 19 from 2.5. (Hint: One method is to convert to a 2 by 2 problem in a and b.)

7. (This computational problem will ask you to open up your favorite computational package, and figure out how to enter a matrix, to matrix multiply, and take the transpose of a matrix. It also asks you to find a pattern.) The 4x4 Pascal Matrix is

$$P = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{pmatrix}.$$

   Look up Pascal's triangle, if you have never heard of it before. A closely related triangle is the lower triangular 4x4 Pascal Matrix

$$L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 3 & 3 & 1 \end{pmatrix}.$$

   Verify on the computer that $P = LL^T$. (L times the transpose of L.) Continue the pattern and create the 5x5 P and the 5x5 L on the computer and verify again that $P = LL^T$ in the 5x5 case.

8. Do Problem 41 from 2.5. Is the answer as nice if you multiply the matrices in the wrong order? (Okay to try this whole problem on paper or on a computer.)

9. (This problem is an investigation on a computer.) Create an identity matrix (for example, in MATLAB, `I=eye(5)`) and a permutation vector (example `p=[3 4 1 2 5]`). Create a permutation matrix by permuting the columns (example `P=I(:,p)`). Compute matrix powers (`P,P^2,P^3,P^4,...`) Which is the smallest positive power that

returns P to the identity? Find five different p's, each with the property that $P^k = I$, for $k = 1, 2, 3, 4, 5$ and k is the smallest such value. Hint: when $k = 1$, the answer is p=[1 2 3 4 5]. When k=5, the answer is p=[2 3 4 5 1]. Now you should find the answers when k=2,3, and 4.

10. The following data is true. The fastest computer in the world, according to the biannual linpack benchmark top 500 figures, is an IBM BLUE GENE at Lawrence Livermore Lab with 1,572,864 core processors.

    It has been clocked at solving a general linear system with $N = 12,681,215$ variables at a rate of 16.32 petaflops/sec or $16.32 \cdot 10^{15}$ floating point operations (basically multiplies and adds) per second.

    Assuming that a solve takes about $(2/3)n^3$ floating point operations, how many hours did it take to solve the system?

    HINT: People usually are willing to wait part of a day, perhaps a whole day, but little more for an answer.

18.06 Wisdom: A lot of math (at least 18.06) is about setting up equations and then doing the same things to both sides in skillful ways.